10 - Probability of AND   [2.6] in Walpole

Example 1. From a standard deck of 52 cards, find the probability of:

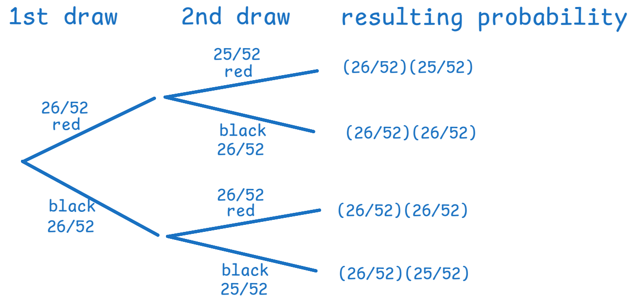
- Drawing two red cards without replacement.

  Answer: P(1st card is red AND 2nd card is red)
  = P(1st card is red) x P(2nd card is red| 1st card is red)
  = (26/52) x (25/51)

- Drawing a red and a black card without replacement **in that order**.

  Answer: P(1st card is red AND 2nd card is black)
  = P(1st card is red) x P(2nd card is black | 1st card is red)
  = (26/52) x (26/51)

- These calculations are summarized in a tree diagram aka a probability tree:



- Drawing two cards of the same color without replacement.

  Answer: From probability tree: P(red & red) + P(black & black) = (26/52)(25/52) + (26/52)(25/52)

Example 2. Probability that some two of six people have the same birth month?

Answer: "Some" = "at least"
P(no two of six people share birth month)
= P(1st person chooses a month to say out loud)
  x P(2nd person's month was not stated by 1st person | 1st person's month was stated)
  x P(3rd person's month was not stated by 1st & 2nd persons | 1st & 2nd person's months were stated)
  x ...
= (12/12)(11/12)(10/12)(9/12)(8/12)(7/12)=22.3%

Take complement: P(some two of six people share birth month) = 100% - 22.3% = 77.7%

Exercises.

1. 12 sweets are in a bag: 7 are lemon, 5 are orange. Two sweets are chosen at random without replacement. Find the probability that: (a) the two sweets are both lemon; (b) the two sweets are both orange; (c) the two sweets are the same flavor; (d) the two sweets are different flavors.

2. In dry weather, the probability of a bus being late is 1/10.
In rainy weather, the probability of a bus being late is 1/4.
In snowy weather, the probability of a bus being late is 2/3.
The probability of dry weather is 3/4.
The probability of wet weather is 1/5.
The probability of snow is 1/20.
(a) Show this information on a tree diagram.
(b) Find the probability that the weather is dry and the bus is on time.
(c) Find the probability that the bus is late.

Answers:
1. (a) 42/132; (b) 20/132;
(c) 62/132; (d) 70/132
2. (b) 0.675; (c) 0.1583

11 - Independent events [2.6] in Walpole

Definition. Events A and B are **independent** if either condition holds:
- P(A and B) = P(A)P(B);
- P(B|A)=P(B) when P(A)≠0; ← since P(A and B) = P(A)P(B|A)
- P(A|B)=P(A) when P(B)≠0. ← "knowledge of event A does not affect P(B)."

Example 3. Are events A and B independent or dependent?

1. Flip a coin twice. A = {1st flip is head}, B = {2nd flip is head}.

   P(B) = 1/2 and P(B|A) = 1/2 so A and B are independent.

2. Roll a dice twice. A = {1st roll is 1, 2, 3}, B = {2nd roll is 2, 4}.

   P(B) = 2/6 and P(B|A) = 2/6 so A and B are independent.

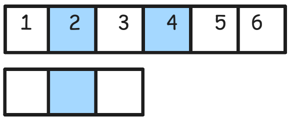
3. Sample 2 cards without replacement. A = {draw a red}, B = {draw a black}.

   P(B) = 26/52 and P(B|A) = 26/51 so A and B are not independent.

4. Sample 2 cards with replacement. A = {draw a red}, B = {draw a black}.

   P(B) = 1/2 and P(B|A) = 1/2 so A and B are independent.

5. Roll a dice once. A = {1st roll is 1, 2, 3}, B = {1st roll is 2, 4}.



Original: P(B)=1/2

Now: P(B|A)=1/2

Example 4. In the contingency table (aka 2-way table), are {>30} and {male} independent?

Answer: No: P(>30|male) = 28/82 = 0.34 but P(>30) = 60/161 = 0.37

	Male	Female	total
under 30	54	47	101
over 30	28	32	60
total	82	79	161

Independence for ≥3 events work the same way. (Ask me about subtleties.)

Example 5. A biased coin is heads with probability p and tails with probability q=1-p. Find the probability of 2 heads in 3 flips?

Answer: In the tree diagram for flipping 3 coins, getting two heads accounts for "3 choose 2" of the resulting leaves, each occurring with probability p^2q. Answer is (3 choose 2) x p^2 x q.

Answer 2: P(HHT) + P(HTH) + P(THH) = P(H)P(H)P(T) + P(H)P(T)P(H) + P(T)P(H)P(H) = 3p^2q.

Exercises.

1. (Network reliability) To connect you to Google, your ISP tries to route you through 3 available routers, with probabilities of success $p_1 = 0.2, p_2 = 0.3, p_3 = 0.8$. Find the probability that at least one router successfully connects you to Google.

2. Every 4th GPU manufactured is tested. The test consists of running four independent tasks and checking the results. Failure rates for the four tasks are, respectively, 0.01, 0.03, 0.02, and 0.01.
(a) Find the probability that a GPU was tested.
(b) Find the probability that a GPU was tested AND failed some test.
(c) Given that a GPU was tested, what is the probability that it failed tasks 2 or 3?
(d) In a sample of 100, how many GPUs would you expect to be rejected?

3. A town has two fire engines operating independently. The probability that a specific engine is available when needed is 0.96. Find the probability that:
(a) neither is available when needed.
(b) a fire engine is available when needed.

4. A fair dice is tossed 3 times. Find the probability of getting exactly two 1's. (Hint: view dice as an unfair coin: "heads" = get 1, "tails" = get 2-6.)

Answers:
1. 0.888
2. (a) 0.25; (b) 0.0171; (c) 0.0494;
(d) 1.7 or 2 after rounding.
3. (a) 0.0016; (b) 0.9984
4. 0.0694